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A Bi-objective Single Period Inventory Problem with Two Stage Inventory Considering Discount and Warehouse Constraint

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Abstract

The classical single-period inventory problem is a tool to find order quantity in a short term that maximizes the expected profit under the probabilistic demand. In this model it is assumed that discount is applied in selling only if any inventory is remained at the end of the period and the news-vendor loses some profit only if order quantity is less than the real demand. In the current survey the utilization of single-period production is considered in which the budget and warehouse space are limited and the end product and raw materials are purchased before the start of selling period and depending on the type of demand during the period the raw materials may transform into end products. The objective of this problem is to find the order quantities of both the raw materials and the end product at the start of the period such that the expected profit and service providing to customers are maximized. Total discount has also been used to purchase raw materials. Due to the complexity of the considered problem, two multi-objective meta-heuristic algorithms of Non-dominated Sorting Genetic Algorithm and Imperialist Competitive Algorithm have been applied to solve the real sized problems in reasonable computational time. Since the performance of meta-heuristic algorithms is significantly influenced by their parameters, Taguchi methodology has been applied to tune the parameters of developed algorithms. Finally the efficiency and applicability of the proposed model and solution methodologies are illustrated through designed experimental problems in various dimensions.

Keywords

Single-period inventory control, multiple raw materials, multi-objective optimization, meta-heuristic algorithms, quantity discount

1. Introduction

The single-period inventory problem is a tool that maximizes the expected profit in the probabilistic demand framework during a period. This problem is used to aid decision making in products regarding fashions, seasonal products, computer products and etc. The problem can also be applied to manage capacity and evaluate booking of orders in service providing industries such as hotels, and airlines. With reduction in the life cycle of products, it has had growing importance in the manufacturing industries. The classical models of inventory control have integrated several various cost criteria and

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service requirements. In these multi-criteria models, the optimizer acts with an optimizer function to simultaneously reach the least maintenance and ordering cost and also the highest level of service providing to customers i.e. it articulates the objective of increasing service providing to customers in the form of shortage cost in order to both be summed up by other inventory costs and also minimized. As the shortage cost is not calculated from accounting documents and bureau, this cost has an approximate nature (Gardner and Dannenbring, 1979) and its utilization disorders the accuracy and practicability of the model. In order to solve the problem, the multifunctional inventory models were proposed where the service providing level to customer is maximized by defining separate objectives and without the estimation of shortage cost.

(Pasandideh et al, 2011) considered the maximization of profit in a single-period single-product inventory problem under the constraints of budget in order to provide raw materials and product at the start of the period. The Lagrangian method was proposed to solve the developed model. The model is the extension of Pasandideh et al which maximizes the profit and service providing level under the constraints of warehouse space and budget and total discount to purchase raw materials at the start of the period. Due to complexity of considered problem, two meta-heuristic algorithms of Non-dominated Sorting Genetic Algorithm and Imperialist Competitive Algorithm have been proposed to solve problems in real sizes and acceptable computational time. Since the performance of meta-heuristic algorithms is significantly influenced by its parameters, Taguchi method has been applied to tune the parameters of developed algorithms. Finally the efficiency and applicability of proposed model and solution methods are demonstrated using designed experimental problems in various dimensions.

The remainder of this paper is organized as follows: In section 2, the literature review and literature survey is provided. The problem definition and description of the developed model is provided in Section 3. The proposed meta-heuristic algorithms are investigated in detail in section 4. In section 5 tuning the parameters is provided. The analysis of the results is also given in section 6. Finally, some conclusions and suggestions for future research are presented in section 7.

2. Literature Review

(Arrow et al.,(1951) and (Kimbal & Morse,(1951) first presented articles about news-vendor problem. In the following years (Hadly & Whitin ,1963) presented a multi-product model. (Chen and Chuang,2000) analyzed the single-period problem along with the shortage level constraint.(Abdel-Malek et al., 2004) considered a single-period inventory problem under the constraint of budget, while the demand distribution function was uniform. They proposed Genetic Algorithm to find the optimum or near optimum solution for general continuous density functions. (Shao and Ji, 2006) investigated a multi-product single-period problem with fuzzy demand problem under the constraint of budget. In order to solve the developed model that maximizes the expected profit of vendor, a hybrid genetic algorithm with fuzzy simulation has been utilized. To name a few further relevant researches, (Zhang ,2010) presented a single-period multi-product inventory problem with discount by suppliers under the constraint of budget and solved it by Lagrangian method. (Shi and Zhang,2010) presented a single-period multi-product model with discount by suppliers and pricing strategy of news-vendor. The aim of this problem was to define optimal quantities and selling price of the products in order to maximize the expected profit under the budget constraint. They both formulated the problem in nonlinear programming and solved it by Lagrangian method. (Shi et al., 2011) considered a problem with cooperative decision making of the retailer and discount by supplier and provided a Lagrangian method to solve the problem.

(Taleizadeh et al .,2011) investigated the single-period problem in a fuzzy environment. In this study, demand for each product was defined as a fuzzy variable and the constraints of service level, batch size, budget, space and high limitation for order had been assumed. The objective was to maximize the expected profit under discount strategy and the proposed model was solved using a combination of fuzzy simulation and five optimization algorithms of Bee Colony Optimization, Harmony Search Algorithm, Particle Swarm Optimization (PSO), Genetic Algorithm and Simulated Annealing (SA). (Taleizadeh et al .,2011) investigated the single-period single-product multi-constraint problem with uncertain demand and incremental discount. In this model, the constraints such as service level, number of orders, warehouse space, and budget allocated and three mathematical models were proposed and the objective was to maximize the profit. (Pasandideh et al., 2011) considered the maximization of profit in a single-period single-product inventory problem under the constraints of budget in order to provide raw materials and product at the start of the period. The Lagrangian method was applied to solve the developed model. (Zhongfeng Qin and Samarjttkar,2013) assumed uncertain demand for product market in a single-period inventory problem where the aim was to maximize the profit.

3. Model Description

3.1 Aassumptions

- The raw materials may be manufactured into the end product during the period.
- There is a one-to-one relation between the end product and the raw materials. It means that one unit of each raw material is consumed in one unit of the finished product.
- The demand for the product is a random variable with a known probability distribution.
- Shortage is allowed.
- A fraction of customers' demands is willing to wait for the product to be manufactured during the period.
- The warehouse space for the end product and raw materials at the start of the period is limited.
- The budget for providing the raw materials and the end product at the start of the period is limited.
- Maintenance cost is allocated to unused raw materials and end product during the period
- In this problem, n types of raw material and only one type of the end products are provided once before the start of the selling period.
- Raw materials and end product will be sold at the end of the period if not used or sold at the start of the period.
- There is a single-period to order raw materials and end product.

3.2. Parameters and decision variables of the model

Indices:

i : Number of raw materials ($i = 1, 2, \dots n$)

k : Number of discount levels ($k = 1, 2, \dots K$)

Parameters:

For $= 1, 2, \dots n$, $k = 1, 2, \dots K$ the parameters of the model are defined as follows:

C_{ik} : The unit cost of providing the ith raw material in discount level of k

t : The unit cost of manufacturing the raw materials into end product

L_i : The discounted price per unit of the ith raw material that is unused during the period and sold at the end of the period

L_m : The discounted price per unit of the end product that is unsold during the period and sold at the end of the period

h_i : The maintenance cost per unit of the ith raw material that is unused at the end of the period

h_m :The maintenance cost per unit of the end product that is unused at the end of the period

r: The selling price per unit of the end product

α: a fraction of the customers that wait for manufacturing raw materials into the end product

π: The shortage cost per unit of the end product
D: The demand random variable of the end product during the period
$f_D(D)$: The probability density function of the demand during the period
f_i: the space required for the ith raw material
S: the space required per unit of the end product
B: The total available budget to provide raw material and manufacture the end product at the start of the period
n: the space required to stock raw materials and end products at the end of the period
u: The profit function
z: service level function
QRU_{ik}: Upper bound for the ith raw material in the kth failing point
QRL_{ik}: Lower bound for the ith raw material in the kth failing point

Decision variables:
QR_i: the quantity of the ith purchased raw material before the start of selling period
QR_m: the least quantity of raw materials $QR_m = \min(QR_1, QR_2, ..., QR_n))$
Q: the quantity of purchased end product before the start of selling period
Q_I: sum of the raw materials at the start of the period
y_{ik}: binary regarding the discount levels

3.3. Problem Formulation
Based on the quantity of the random demand, the following three cases may occur:

Case 1: The demand is less than or equal to the quantity of the end product at the start of the period. In this case, no production is required and raw materials remain untouched, i.e., $0 \leq D \leq Q$. The profit function is also obtained as the following:
here

$$U_1(QR_i, Q, D) = r(D) + L_m(Q - D) + \sum_{i=1}^{n} L_i(QR_i) - t(Q) - h_m(Q - D)$$
$$- \sum_{i=1}^{n}\sum_{k=1}^{k} C_{ik} QR_i Y_{ik} - \sum_{i=1}^{k} h_i(QR_i) - \sum_{i=1}^{n}\sum_{k=1}^{k} C_{ik} Y_{ik} Q \tag{1}$$

- $U_1(QR_i, Q, D)$ is the profit function of the first case
- $r(D)$ is the revenue obtained by selling the end product during the period
- $\sum_{i=1}^{n} L_i(QR_i)$ is the revenue obtained by selling the remaining units of the raw materials at the end of the period at a discounted price
- $L_m(Q-D)$ is the revenue obtained by selling the remaining units of the end product at the end of the period at a discounted price
- $\sum_{i=1}^{n}\sum_{k=1}^{K} C_{ik} QR_i y_{ik}$ is the total cost of providing the raw materials at the start of the period
- $h_m(Q-D)$ is the maintenance cost of the remaining units of the end product at the end of the period
- $\sum_{i=1}^{n} h_i(QR_i)$ is the maintenance cost of the remaining units of the raw materials at the end of the period
- $t(Q)$ is the production cost of the end products from raw materials at the start of the period

- $\sum_{i=1}^{n}\sum_{k=1}^{K} C_{ik}\, y_{ik} Q$ is the cost of providing the utilized raw materials to produce end product during the period

Case 2: In the second case, the demand is greater than the end product quantity at the start of the period i.e. $\alpha(D-Q) \le QR_m,\ D > Q$. In this case the profit function is obtained as the following:

$$U_2(QR_i, Q, D) = r(Q) + r(D-Q)(\alpha) + \sum_{i=1}^{n} L_i\,(QR_i - \alpha(D-Q)) - t(Q) - t\alpha(D-Q)$$
$$-\sum_{i=1}^{n}\sum_{k=1}^{K} C_{ik}\, Y_{ik}\, Q - (1-\alpha)(D-Q)\pi - \sum_{i=1}^{n} h_i\,(QR_i - \alpha(D-Q)) - \sum_{i=1}^{n}\sum_{k=1}^{k} C_{ik} QR_i Y_{ik} \tag{2}$$

where

- $U_2(QR_i, Q, D)$ is the profit function of the second case
- $r(Q + \alpha(Q - D))$ is the revenue obtained by selling all the units of the end product and also selling a percentage of the demands that are manufactured by raw materials during the period.
- $\sum_{i=1}^{n} L_i\,(QR_i - \alpha(Q-D))$ is the income obtained by selling the remaining units of the raw materials at discounted prices at the end of the period.
- $T\alpha(D\text{-}Q)$ is the cost of manufacturing the raw materials into the end product during the period (for customers that wait)
- $(1\text{-}\alpha)(D\text{-}Q)\pi$ is the shortage cost of unsatisfied demand of the end product
- $\sum_{i=1}^{n} h_i\,(QR_i - \alpha(D-Q))$ is the maintenance cost of the remaining units of raw materials at the end of the period

Case 3: In the third case the demand is greater than the end product quantity at the at the start of the period and $\alpha(D-Q) > QR_m,\ D > Q$ or $D > Q + \frac{QR_m}{\alpha}$. In this case the profit function is obtained as the following:

$$U_3(QR_i, Q, i) = r(Q) + r(QR_m) + \sum_{i=1}^{n} L_i\,(QR_i - QR_m) - (t + \sum_{i=1}^{n}\sum_{k=1}^{k} C_{ik} Y_{ik} Q) - t(QR_m)$$
$$-\pi(1-\alpha)(D - Q - QR_m) - \sum_{i=1}^{n} h_i\,(QR_i - QR_m) - \sum_{i=1}^{n}\sum_{k=1}^{k} C_{ik}\, QR_i Y_{ik} \tag{3}$$

where

- $U3(QR_i,Q,D)$ is the profit function of the third case
- $r(Q)$ is the revenue obtained by selling all the units of the end product that are available at the start of the period
- $r(QR_m)$ is the income obtained by selling the end products manufactured during the period
- $\sum_{i=1}^{n} L_i\,(QR_i - QR_m)$ is the revenue obtained by selling the remaining units of the raw materials at the end of period with a discounted price
- $t(QR_m)$ is the cost of manufacturing the raw materials into the end product during the period
- $\pi(1\text{-}\alpha)(D\text{-}Q\text{-}QR_m)$ is the shortage cost of unsatisfied demand of the end product

- $\sum_{i=1}^{n} h_i (QR_i - QR_m)$ is the maintenance cost of the remaining units of the raw materials at the end of period

3.3.1. calculation of the expected profit during the period

The profit function of subsection 3-3-1 is obtained as follows:

$$U(QR_i, Q, D) = \begin{cases} U_1(QR_i, Q, D) & if \quad D \le Q \\ U_2(QR_i, Q, D) & if \quad Q < D \le Q + \dfrac{QR_m}{\alpha} \\ U_3(QR_i, Q, D) & if \quad D > Q + \dfrac{QR_m}{\alpha} \end{cases} \tag{4}$$

The expected profit ($\overline{u}$) is calculated as follows:

$$\overline{u} = \int_0^{\infty} U(QR_i, Q, D).\, f_D(d).dd \tag{5}$$

According to Eq. (6), the expected profit transforms as follows:

$$\overline{u} = \int_0^{Q} U_1.\, f_D(d).dd + \int_Q^{Q+\frac{QR_m}{\alpha}} U_2.\, f_D(d).dd + \int_{Q+\frac{QR_m}{\alpha}}^{\infty} U_3.\, f_D(d).dd \tag{6}$$

3.4. Problem Modeling

$$Max.\overline{u} = \int_0^{Q} U_1.\, f_D(d).dd + \int_Q^{Q+\frac{QR_m}{\alpha}} U_2.\, f_D(d).dd + \int_{Q+\frac{QR_m}{\alpha}}^{\infty} U_3.\, f_D(d).dd \tag{7}$$

$$Max.\overline{z} = P(D \le Q) = \int_0^{Q} f_D(d)dd \tag{8}$$

s.t:

$$\sum_{i=1}^{n} f_i.QR_i + S.Q \le n; \tag{9}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{k} C_{ik}.y_{ik}.QR_i + t.Q + Q\sum_{i=1}^{n}\sum_{k=1}^{k} C_{ik}.y_{ik} \le B; \tag{10}$$

$$QR_i < QRU_{ik}.y_{ik}; \quad \forall i,k \tag{11}$$

$$QR_i \ge QRL_{ik-1}.y_{ik}; \quad \forall i,k \tag{12}$$

$$\sum_{k=1}^{k} y_{ik} = 1; \quad \forall i \tag{13}$$

$$QR_m,\ QR_i,\ Q \ge 0; \quad \forall i \tag{14}$$

$$y_{i0} = 0; \qquad \forall i \tag{15}$$

$$y_{ik} \in \{0,1\}; \qquad \forall i,k \tag{16}$$

- The objective function (7) is the maximization of the expected profit during the period and the objective function (8) is the maximization of service providing level.
- Constraint (9) shows the warehouse capacity limitation at the start of the period in order to save end products and raw materials before the start of the selling period.
- Constraint (10) shows the limitation of the available budget at the start of the period in order to provide raw materials and end products.
- Constraints (11) and (12) are the intervals of proposed discount by suppliers to provide raw materials at the start of the period
- Constraint (13) assures that the purchase is performed only in one price level.
- Constraints (14) - (16) show the domain of the decision variables.

4. Proposed Meta-Heuristic Algorithms

Since Burke et al (2008) proved the problems with quantity discount are considered to be NP-hard, two meta-heuristic algorithms of Non-dominated Sorting Genetic Algorithm and Imperialist Competitive Algorithm have been applied to solve the problem.

4.1. Non-dominated Sorting Genetic Algorithm

Non-dominated Sorting Genetic Algorithm was introduced by Deb et al (2000). The steps of the algorithm are described in details in the following:

4.1.1. Initial Population

In order to initiate the algorithm, it is required to define the initial population, mutation operator probability, crossover breeding operator probability, generation production, and control parameter of Elitism Approach. The initial population was produced randomly.

4.1.2. Chromosome Evaluation

In this article, each chromosome is a random vector. Each solution is a linear array of 1*(raw materials quantity+1) dimension in which the last index (gene) represents the end product quantity and the genes preceding represent raw materials quantity. Every index of the chromosome is a random value in demand interval.

67.38	172.36	189.12	72.96	138.64

Fig. 1: chromosome configuration

Since different constraints may result in infeasible solutions, returning strategy has been used to confront the problem constraints.

4.1.1.3. Non-dominated Sorting and Crowding Distance

In this subsection, the population sorting is performed in two steps:

I. Fast Non-dominated sorting

In order to sort out a population of size n according to the level of non-domination, each solution is compared with every other solution in the population to determine whether the solution will be dominated or not. Finally there would be several solutions which will not dominate others or become dominated by them. Hence these solutions are the first boundary (front) of the non-dominated boundaries. In order to determine the solutions on hand in the next fronts, the solutions in the first front are ignored temporarily and the process is repeated again until all the solutions enter the non-dominated boundaries.

II. Crowding Distance

In order to estimate the density surrounding a specific solution of the population, we calculate the average distance of this solution from two adjacent solutions on both sides according to target values. This value is called the crowding distance.

4.1.1.4. Parents

In this subsection, the parents which have experienced non-dominated sorting and crowding distance, are maintained in order to apply mutation operator and crossover breeding operator according to Selection Strategy.

4.1.1.5. Selection Strategy

In this subsection, Crowded Tournament Selection Operator is applied to select parents in order to apply mutation operator and crossover breeding operator i.e. a solution is better only if it has a better rank and in case of having an equal rank, it should has more crowding space.

4.1.1.6. Crossover Breeding Operator

The proposed Crossover breeding operator is applied on the selected parents with crossover probability of P_c. An off-spring is produced from the two parents by single-point crossover breeding operator.

4.1.1.7. Mutation Operator

The solution spaces that are not discovered by the crossover operator are found using the mutation operator. In this paper, the Swap Mutation Operator was used.

4.1.1.8. Off-spring Evaluation

In this subsection all the off-springs produced by mutation and crossover breeding operators are evaluated.

4.1.1.9. combination of parents and off-spring populations

In this subsection the population of parents and off-springs are combined and a population of twice the initial population's size is created. The aim of this combination is not to eliminate the better solutions between the parents and off-springs populations.

4.1.1.10. Non-dominated Sorting and Crowding Distance

In this subsection, the Non-dominated Sorting and Crowding Distance explained in the previous subsection are performed on the parents and off-springs population.

4.1.1.11. population sorting and selection of N chromosomes

Initially the population members of each boundary are sorted based on the crowding distance and then sorted based on the non-domination. This leads to the sorting of the sorted population based on the non-dominated boundary and crowding distance.

4.1.1.12. Termination Condition

The algorithm is terminated after a pre-selected number of generations or a maximum number of iterations.

4.2. Multi-Objective Imperialist Competitive Algorithm

Imperialist Competitive Algorithm (ICA) was first presented by Atashpaz-Gargari and Lucas (2007). The algorithm initiates with an initial population of countries. It follows a socio political process called the Imperialist competition. Among these, the best countries in the population are selected to be imperialists and the rest form the colonies of these imperialist. An imperialist with its colonies form an "empire". In the following subsections, the steps of the proposed multi-objective Imperialist Competitive Algorithm for solving the problem are described in details.

4.2.1.Generating Initial Countries

In this subsection, an array of decision variable values is formed to determine the optimal values in the search area. In an optimization problem of N_{var} dimension, a country is a $1 * N_{var}$ array which is defined as:

$$Country = \{p_1, p_2, \ldots, p_{N_{\mathrm{var}}}\} \tag{17}$$

where P_i is a variable which presents the socio political features of the country such as culture, language, economic structure and etc. After generating the initial countries' population and evaluating the fitness value of each solution, the solutions are sorted in various fronts according to the non-dominated technique in the form of Pareto criterion.

4.2.2. Imperialist selection and forming the empire

After generating the countries and calculation of non-dominated solutions, the best countries in the population are selected to be imperialists (N_{imp}) and the rest form the colonies of these imperialists (N_{col}). The Imperialist with the highest power has more chance of possessing more colonies.

4.2.3. The normalized cost of the i^{th} objective function for the n^{th} imperialist

Each imperialist has a cost which directly influences its colonies. In order to calculate the cost of an imperialist, the normalized cost of the ith objective function for the nth imperialist is computed as Eq. (18):

$$C_{i,n} = \frac{\left| f_{k,n}^{p} - f_{i}^{p,best} \right|}{f_{i,total}^{p,max} - f_{i,total}^{p,min}} \tag{18}$$

where $C_{i,n}$ is the normalized cost of the ith objective function for the nth imperialist and $f_{i,total}^{p,min}$, $f_{i,total}^{p,max}$, $f_{i}^{p,best}$ are the maximum and minimum size of the objective function in each iteration respectively.

4.2.4. Normalized Cost of the nth Imperialist

$$Total\ Cost_n = \sum_{i=1}^{r} C_{i,n} \tag{19}$$

4.2.5. Imperialist Power

$$power_n = \left| C_n \Big/ \sum_{i=1}^{N_{imp}} C_i \right| \tag{20}$$

4.2.6. Assignment of Colonies to the Imperialists

The Normalized power of an imperialist is the ratio of the colonies administered by that imperialist. Therefore the initial number of colonies of an imperialist is according to Eq. (21).

$$NCOL_n = round\{power_n \times N_{col}\} \tag{21}$$

where $NCOL_n$ is the initial number of colonies of an imperialist and N_{col} is the number of total colonies in the population of the initial countries.

4.2.7. Total Power of an Empire

The Power of an Empire is calculated as the summation of the imperialist power and a percent of total power of all of its colonies.

4-2-8- Movement of the colonies towards their Imperialists (Identification)

Considering a country's representation in an optimization problem, this central government was trying to take possession of a colony in various socio political dimensions through attraction policy. The imperialist country attracts a colony in culture and language directions. The colony moves directly toward the imperialist to a new position by generating a random variable (x) which acts as the uniform distribution (or whatever the suitable distribution) Where the distance between a colony and an imperialist is indicated by parameter d, i.e.

$$x \sim U(0,\ \beta \times d) \tag{22}$$

where β is a value greater than 1 and close to 2. β=2 could be a good choice. β>1 means that a colony could move toward an imperialist from different directions.

4.2.9. Information Exchange among colonies

The colonies of each empire exchange their information through crossover operation in order to improve themselves.

4.2.10. Paired Competition Operator

The Paired Tournament selection method is used to select solutions in mutation and crossover breeding operators.

4.2.11. Crossover among colonies

In this subsection, a specified number of the weakest countries of each empire are obtained by Roulette Wheel Selection. Then according to the multiplication of number of countries of each empire by crossover operator rate and the shortage round of the obtained value, the number of the relocations among the off-springs of the operator and the weakest countries is obtained as well.

4.2.12. Revolution

Each iteration, a number of colonies of each empire is selected at a revolution ratio by Paired Competition Operator. The revolutions are implemented on some countries.

4.2.13. Updating the colonies

The initial population of colonies, the population of identification, the population of information exchange and, the population of revolution are merged in every empire. Then the non-dominated solutions are introduced into the archive. Also the best solutions of the merged population are selected based on the domination criterion and crowding space based with similar number of colonies of the empire and the rest are omitted.

4.2.14. Updating the Archive

The merged population is sorted according to domination and crowding space and the non-dominated solutions of the primary front enter the archive.

4.2.15. Relocation of the colony and the imperialist

In this step, the total cost of all the imperialists is updated. Then the imperialist and population of its colonies are merged in every empire and sorted according to domination criterion and crowding space. Finally the best solution of the population is selected as the imperialist.

4.2.16. Imperialists competition

The empires which are not able to increase their power and lose the competition will be gradually out of the competition. This imperialistic competition gradually brings about one or more of the weakest colonies and allowing the empires to compete for having the chosen colonies. The colonies will not necessarily be possessed by the most powerful empires but more powerful empires have a more chance of being selected.

4.2.17. The decline of the weakest empires

An empire will be out of the imperialistic competition In the Imperialists competition algorithm when it loses all of its colonies.

5. Tuning the Parameters

The Taguchi (Taguchi, 1986) is a method of Design of Experiments presented in World War II by someone with this name. Also, in this method the variance of experimental data with the variance of response variables is investigated. In order to facilitate the investigation, the Signal-to-Noise ratio concept (S/N) is used. In order to increase the efficiency of the proposed meta-heuristic algorithms, we will be tuning some input parameters of the algorithms. The parameters levels of Non-dominated Sorting Genetic Algorithm and Imperialist Competitive Algorithm are presented in tables 1 and 2 respectively.

Table 1. Parameters levels of Non-dominated Sorting Genetic Algorithm

Factors	Levels		
	Level 1	Level 2	Level 3
Popsize (A)	100	150	200
Crossover rate (B)	0.8	0.85	0.9
Mutation rate (C)	0.03	0.05	0.10
Number of generations (D)	70	90	120

The proposed meta-heuristic algorithms are performed for each Taguchi Experiment. Then the Signal-to-Noise ratio is calculated in Minitab Software version 14/1(2003). The optimal level for the NSGA-II algorithm is (*A=200,B=0.85، C=0.05،D=70*) and for the MOICA algorithm is *(A=150 ،B=20 ، C=0.05 ،D=110*).

6. Analysis of the results

In this section, the results obtained by both Non-dominated Sorting Genetic Algorithm and Imperialist Competitive Algorithm have been investigated from three viewpoints as following: comparison of algorithms' performance based on the evaluation criteria of multi-objective algorithms, statistical analysis and comparison of algorithms' performance based on Multi-Attribute Decision Making technique. It is noteworthy that the results of the criteria obtained by GAMS Software in solving four problems of small size with the objective functions integrated by epsilon constraint were the same as the results obtained by running the algorithms.

Table 2. Parameters levels of Imperialist Competitive Algorithm

Factors	Levels		
	Level 1	Level 2	Level 3
Number of countries (A)	100	150	200
Number of imperialists (B)	10	15	20
Revolution ratio (C)	0.05	0.15	0.30
Number of generations (D)	80	110	150

6.1. Numerical Example

Since no problem is presented for the mathematical model of this research in the literature, 16 experimental problems with raw materials quantity (n) and number of discount points (k) are presented in table 3. Also the input parameters values for the examples are according to table 4.

Table 3. Designed experimental problems

Problem NO.	Problem Size	Raw material	Discount point
1	Small	**3**	2
2		**3**	3
3		**5**	4
4		**7**	4
5		**10**	2
6		**15**	3
7		**20**	4
8		**100**	3
9	Medium	**200**	4
10		**300**	5
11		**500**	4
12		**600**	2
13		**800**	3
14	Large	**1000**	3
15		**1500**	4
16		**2000**	4

Table 4. Input parameters values

Parameters	Values
(D) Demand	U[100 200]
(t) The production cost per unit of the product	U[1000 2000]
(h_m) The maintenance cost per unit of the product	U[300 400]
(h_i) The maintenance cost per unit of the ith raw material	U[100 250]
(π) The shortage cost per unit of the product	U[400 600]
(C_{ik}) The cost of purchasing the ith raw material in the first price level	U[350 850]
(C'_{ik}) The reduction of cost of purchasing the ith raw material in higher price levels	U[5 30]
(α) A fraction of the customers waiting for production	0.3
(B) The available budget at the start of the period	2500000
(n) The warehouse space available at the start of the period	20000
(L_i) The reduction in the price of selling the ith raw material at the end of the period	U[5 25]
(L_m) The reduction in the price of selling the end product at the end of the period	U[5 30]
(f_i) The space required per unit of the ith raw material	U[5 10]
(S) The space required per unit of the end product	U[40 50]

6.2. comparison of the algorithms' performance based on the evaluation criteria of multi-objective algorithms

- **Quality Metric (QM)**

The percentage of non-dominated solutions pertaining to each algorithm shows its quality. The algorithm which has the highest number of the non-dominated Pareto solutions has more quality metric and is more desirable.

- **Mean Ideal Distance Metric (MID)**

This metric represents fronts distance from the best solution of the population

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- **The Rate of Achievement to Objectives Simultaneously (RAS)**

- **Spacing Metric (SM)**

This metric was proposed by Schott (1999). Using this metric, the uniformity of distribution of non-dominated solutions is obtained. The less the SM is, the better is the algorithm.

- **Computational time of the algorithm**

6.3. Statistical Comparison of results

P-values of variance analysis and approval or refusal of null hypothesis are presented in table 8. The null hypothesis is the equality of means of evaluation criteria in both algorithms at the 0.95 confidence level. If the P-Value is less than 1-0.95=0.5, the null hypothesis is refused and we conclude that there is a meaningful difference between the evaluation criteria of both algorithms' performance and vice versa. The results of criteria are normally distributed. The results of variance analysis imply that both algorithms are almost competitive in terms of RAS and Time criteria and there are meaningful differences between the algorithms' performance in the rest of the criteria. In order to calculate the weight of the metrics we have used Antropy technique showing that the proposed MOICA significantly works better than the proposed NSGA-II.

Table 5. Computational results of non-dominated sorting algorithm

Problem NO.	Criterion				
	MID	QM	RAS	SM	Time(sec)
1	0.7154	0.6759	0.8202	0.8243	119.40
2	0.8146	0.4762	0.9325	0.6226	119.42
3	0.6075	0.1667	0.7842	0.4954	118.75
4	0.6763	0.3182	0.7522	0.7780	119.45
5	0.7137	0.5083	0.7896	0.7826	120.04
6	0.8104	0.3519	0.9052	0.6702	120.93
7	0.7606	0.2889	0.8375	0.6194	117.12
8	0.7151	0.5000	0.7960	0.5761	117.58
9	0.8195	0.5000	0.9301	0.5364	141.59
10	0.8192	0.4071	0.8928	0.5143	152.12
11	0.8360	0.0833	0.9095	0.8310	158.60
12	0.7951	0.4947	0.8715	0.7919	150.43
13	0.7176	0.5595	0.7934	0.6124	195.41
14	0.7963	0.3132	0.9017	0.5262	215.54
15	0.7872	0.4929	0.8997	0.6408	248.89
16	0.8658	0.2639	0.9695	0.4889	249.82
Mean	0.7712	0.40004	0.8616	0.6444	154.067

Table 6. Computational results of Multi-Objective Imperialist Competitive Algorithm

Problem NO.	Criterion				
	MID	QM	RAS	SM	Time(sec)
1	0.7734	0.3214	0.8202	0.3975	81.37
2	0.8513	0.5238	0.8731	0.5259	82.92
3	0.7418	0.8333	0.7880	0.6446	80.37
4	0.8310	0.6818	0.9024	0.5260	79.92
5	0.9218	0.4917	0.9841	0.2446	77.22
6	0.8871	0.6481	0.9554	0.4454	84.46
7	0.7187	0.7111	0.7748	0.5844	79.46
8	0.9153	0.5000	0.9341	0.3120	88.08
9	0.7893	0.5000	0.8475	0.4442	108.51
10	0.9182	0.5929	0.0449	0.5478	115.64
11	0.7660	0.9167	0.8523	0.6349	141.73
12	0.8730	0.5053	0.9612	0.2885	132.16
13	0.8314	0.4405	0.9292	0.4400	180.05
14	0.8354	0.6868	0.9395	0.4038	206.50
15	0.7014	0.5071	0.7739	0.4547	253.09
16	0.8508	0.7361	0.9512	0.6255	277.50
Mean	0.8253	0.5999	0.8957	0.4699	129.33

Table 7. The results of variance analysis of comparison criteria of algorithms

Criterion	P-Value	Test result
MID	**0.024**	null hypothesis refusal
QM	**0.001**	null hypothesis refusal
RAS	**0.198**	null hypothesis approval
SM	**0.000**	null hypothesis refusal
Time	**0.231**	null hypothesis approval

6.4. Comparison of algorithms' performance based on the Multi-Attribute Decision Making Techniques (MADM)

In order to choose a more suitable algorithm by TOPSIS method, the information of table 8 being a decision making matrix has been used.

Table 8. Information required to form the decision making matrix

Algorithm	Criterion						
	Average profit	Average Service level	MID	QM	RAS	SM	Time
NSGA-II Algorithm	30953935	0.7948	0.7712	0.4	0.8616	0.6444	154.07
MOICA Algorithm	22166843	0.8031	0.8253	0.6	0.8957	0.47	129.33

7. Conclusion and Suggestions for Future Research

In this research, first a total view of inventory control and single-period inventory and then a comprehensive review in the findings was performed and then a single-period inventory problem has been developed, in which there is a single product and several raw material at the start of the period. It was assumed that the remaining product and raw materials are sold at a discounted quantity at the end of the period. Also a total discount is given by the suppliers of the raw materials. In order to make the problem reliable and closer to reality, the constraints of budget and warehouse space are assumed to provide product and raw materials at the start of the period. The problem has been formulated in the non-linear programming together with multi-objective digital number which maximizes the expected profit and sevice providing level to customers simultaneously. Due to complexity of the proposed NP-hard problem in the literature, the meta-heuristic algorithms of Non-dominated Sorting Genetic Algorithm and multi-objective Imperialist Competitive Algorithm have been developed to solve the real sized problems in reasonable computational time. Since the performance of meta-heuristic algorithms is highly influenced by its input parameters, Taguchi methodology has been applied to tune the parameters of proposed meta-heuristic algorithms. Finally the efficiency and applicability of the proposed model and solution methodologies are illustrated through designed experimental problems in various dimensions and the comparison criteria of multi-objective problems, statistical analyses and Multi-Attribute Decision Making techniques have been utilized to choose the most efficient solution methods. Considering the extension of the scope investigated, we suggest the followings for future research:

- Some of the parameters of the model may be considered fuzzy.

- In cases in which there is not a one-to-one relation between the end product and the raw materials, the problem may be remodeled.
- The model of this research may be extended to a multi-product model.
- When only the mean and the variance of the random demand are known and the demand is freely distributed, the problem may be re-formulated.
- The inspection for raw materials and product at the start of the period could be considered as well.
- The problem could be remodeled considering the losses and reworking.
- The constraints of weight of materials and products, maximum production and etc, could be assumed in addition to the constraints of budget and space.

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