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Evaluating the rotation of the main stress and strain axes and the lack of their rotation coaxiality using Multi laminate theory

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ABSTRACT

The soil behavior depends on various factors such as stress path during loading. Most soil behavior models are independent of stress and strain and are therefore unable to predict soil behavior while rotating the main stress and strain axes. According to Multi-laminate theory, in pasty elastic behavior due to the necessity of reflecting the development of stress and strain and its directions on plates passing through a point, the information of these changes remains intact and the behavioral changes will occur at least on one of these plates, which will be reflected in the mechanical behavior of a point and its effects will be seen. In this paper, the rotational effects of the main stress and strain axes are predicted based on the Multi-laminate pasty elastic pattern and compared with the experimental results that Provide adapted results.

1. INTRODUCTION

Soil materials behave differently. The main factors for these differences are their Multiphase and deformation. Soil deformation depends on many factors such as particle shape and size, porosity, adhesion and friction of grains, moisture content, saturation percentage, drainage, lateral bend, path and stress history, loading rate and homogeneity of materials. For this reason, it is difficult to predict soil behavior or deformation. The lack of Coaxiality, Asymmetricity in the main stress and the main strain developing path are defined in geotechnical engineering. This important phenomenon is observed in both engineering issues and laboratory results of direct cutting experiments and Hollow Cylinder Test [1-3].

The numerical analysis by Yu, Yuan, Yu, and Yang showed that the lack of coaxiality in granular soil has important implications for geotechnical design. They concluded that designing surface foundations without considering coaxiality could be uncertain. For the first time in 1967, Roscoe reported a lack of coaxiality for the main stresses and the strain direction for simple shear tests.

Using simple shear experiments, Arthur and Wong explained that in sand samples under the continuous rotation of the main stress axis, the deviation between the directions of the development of the main stress and the development of the main strain can be more than 30 degrees. Lack of coaxiality refers to the difference in direction between the Axes of main stress and the main strain development. By considering the same axes and

applying the main stresses (σ_{ij}) and the main strain ($d\varepsilon_{ij}$), the tensors $\overline{\sigma}_{ij}$ and $d\overline{\varepsilon}_{ij}$ can be obtained with appropriate rotation of tensors σ_{ij} and $d\varepsilon_{ij}$ [4]:

$$\overline{\sigma}_{ij} = A_{ik} \sigma_{kl} A_{lj} \tag{1}$$

$$d\overline{\varepsilon}_{ij} = B_{ik}\sigma_{kl}B_{lj} \tag{2}$$

Where σ_{ij} is the stress tensor in the desired direction.

 $d\varepsilon_{ij}$ is the developmental strain tensor in the desired direction, A_{ij} and B_{ij} are rotational tensors for the main stresses and developmental strains. Thus if $A_{ij} \neq B_{ij}$, the lack of coaxiality of the tensors $\overline{\sigma}_{ij}$ and $d\overline{\varepsilon}_{ij}$ is observed. The purpose of this paper is to explore the lack of coaxiality of main stresses and the developmental strains using multi laminate theory.

In this theory, the elastoplastic model is used with the Hardening law. One of the benefits of this theory is the dependence of soil behavior on different loading directions and the possibility of using homogeneous soil in different directions. This model is also able to predict the fracture plate under different loads.

1.1. Multi laminate Theory

The Multi laminate theory, is a numerical approximation of some of the specific physical properties contained in the volume of an environment. This is done numerically by multiplying the physical values of the sample points by their weighting coefficients and taking the sum of these values as the properties desired for that environment. The numerical concept of multi laminate theory is based on the numerical integration of a mathematical function on the sphere surface with a single radius. In numerical integration, the surface of a hypothetical sphere with a single radius can be approximated by the number of flat laminates tangent to the surface of the sphere. As such, each of these laminates

has a contact point with the surface of the sphere, and by limiting these laminates, the number of base points can also be defined. [5, 6]. According to the theory of multi laminate, the soil behavior is determined on the basis of slipping and perpendicular deformation to the boundaries of soil grains. In the multi laminate model, the sum of soil deformations is assumed to be due to the contribution of the surface contact slip of the various blocks. When a polyhedral block is subjected to a small shear stress, elastic shear deformation occurs.

As these shear stresses increase and reach a certain extent, these blocks begin to slip along and parallel to the laminates, called slip planes, and become more deformed. These deformations caused by slip are the same as plastic deformations. The total shear deformation at any time is the sum of the elastic shear deformations in the polyhedral block and the deformations caused by the slip of adjacent parts. Figure 1 shows 13 sample planes in the same cubes.



Figure 1. The position of the thirteen planes [6]

In a continuity area, the relationship between stress and strain is as follows:

$d\varepsilon = d\varepsilon^e + d\varepsilon^p$	(3)
$d\varepsilon^e = C^e . d\sigma'$	(4)
$d\varepsilon^p = C^p. d\sigma'$	(5)

Where C^{p} and C^{e} are soft plastic and elastic matrices. In this model, assuming the graininess of the sand, the Mohr-Coulomb yield criterion is used as the final yield limit. Therefore, the yield function in shear stress space and perpendicular to the plane is defined as follows. [7, 8]

$$F_i(\sigma'_{ni},\tau_i,\eta_i) = \tau_i - \eta_i \sigma'_{ni} \tag{6}$$

Where τ and σ'_n are the shear stress and the effective stress perpendicular to the fracture plane respectively, and ϕ'_{f} is the maximum internal friction angle.

 $\eta_i = \tan \alpha_i$ is hardness analog. The pasty potential function in this model is the potential function presented by Feda. This function is based on minimizing the potential energy level.

$$Q_i(\sigma'_{n_i},\tau_i) = \tau_i - \eta_c \sigma'_{n_i} \ln\left(\frac{\sigma'_{n_i}}{\sigma'_{n_o}}\right)$$
(7)

According to the law of flow and the condition of compatibility we can write:

$$d\varepsilon^{p} = \lambda_{i} \left(\frac{\partial Q_{i}}{\partial \sigma'} \right) = \frac{1}{H_{i}^{p}} \cdot \left(\frac{\partial F_{i}}{\partial \sigma'} \right) \cdot \left(\frac{\partial Q_{i}}{\partial \sigma'} \right)^{T} \cdot d\sigma'$$
(8)

The coefficient of plane hardening is defined as follows:

$$H_i^p = -\frac{\partial F_i}{\partial \eta_i} \cdot \frac{A_i \tan \phi'_f}{\left(A_i + \varepsilon_i^{ps}\right)^2} \cdot \frac{\partial Q_i}{\partial \tau_i}$$
(9)

1.2. Strength Ellipsoid

Material breakdown occurs when the stress vector on one or more plates reaches the breakdown cover. The failure criterion can be defined differently for each plane.

But on every plane, this amount should not exceed $\tan \phi$. For an ideal granular environment, the breakage coating can be a spherical coating that provides equal resistance in all directions. But it is more appropriate to use an elliptical coating to influence the soil structure due to the deposition plane. Any change in the main stress axis results in the formation of a series of different resistances on different planes. This ellipse that is perpendicular to the deposition plane is defined as follows [9]:

$$\frac{z^2}{A^2} + \frac{x^2}{B^2} + \frac{y^2}{C^2} - 1 = 0$$
(10)

Where A, B, and C are elliptic diameters along x, y, and z. In general mode, B = C. Therefore, to apply this method, the results of soil behavior in two directions of 0 °(triaxial stress standard) and 90 ° (triaxial strain standard) are required. The $\tan \phi'$ value in the direction of (l'_i, m'_i, n'_i) is calculated as follows:

$$\tan \phi' = \frac{A.B}{\sqrt{(l_i'^2 + m_i'^2)A + n_i'^2 B}}$$
(11)

 l_i, m_i, n_i are directions of stress vector cosines.

2. ASSIGNING THE MODEL TO THE SOIL SAMPLE AND DEFINING THE MODEL PARAMETERS

The model presented in this thesis has the following invariants:

1. Elastic parameters including E, v

2. breakdown Parameters of Mohr Coulomb Including C, ϕ'_{f}

3. Critical state parameters η_c

4. Soil behavior parameter A_0

5. Parameter of hardening or slope of initial yield line

$$\eta_0$$

In this paper, the experimental results of Jung Wang are used on Portaway sand [10]. The results of the model calibration with the experimental results of the Portaway sand are as follows:

Table 1. 1 atameters used in simulating the behavior of 1 of taway sand					
Elastic parameters	breakdown State Parameters	Critical State Parameters	Soil behavior parameter	Hardening parameter	
v = 0.19 $E = 2290kPa$	C = 0 $\phi = 19^{\circ}$	$\eta_c = 0.85\eta_i$	$A_0 = 0.0017$	$\eta_0 = 0.01$	

Table 1. Parameters used in simulating the behavior of Portaway sand

The results of simulating the behavior of the Portaway sand (90%) are shown in Figure 2. In this simulation, confinement stress equal to 600 kPa was used. In this case, the soil will shrink slightly and then expand rapidly.



Figure 2. Results of simulating the behavior of the Portaway sand

3. MODEL RESULTS

In this section, a set of simulation results on Portaway sand behavior under unilateral loading in a drained state is presented to investigate the lack of coaxiality behavior of this soil. First, all samples are consolidated in shape of isotropy to a limiting pressure of 600 kPa. In this type of loading, the deviator stress (q) is applied to the specimens unilaterally until they reach breakdown with the stress paths described below. Since the HCA cannot rotate the main axes at zero bends, the samples are first subjected to the 8 kPa deviator stress and then the main axes are rotated to the required angle.

It should be noted that in all these simulations the mean (P) stress value is kept constant during the deviator stress. After this stage, the deviator stress increases at the desired angle until breakdown occurs. The results of the model in unilateral loading at 90 and 0.30,45.60 degrees are given below.



Figure 3. The Simulation results of the behavior of Portaway sand in the drained state for $\alpha = 0^{\circ}$ (a) Deviator stress-strain components (b) maximum main strain-deviator stress (c) bulk strain -deviator strain (d) lack of coaxiality for stress direction and strain development $\alpha_{d\varepsilon}$



Figure 4. Simulated results of the behavior of Portaway sand in the drained state for $\alpha = 30^{\circ}$ (a) deviator stress-strain components (b) deviator stress-maximum main strain (c) bulk strain-deviator strain (d) lack of coaxiality for stress direction and strain development



Figure 5. Simulated results of the behavior of Portaway sand in the drained state for $\alpha = 45^{\circ}$ (a) deviator stress-strain components (b) deviator stress-maximum main strain (c) bulk strain-deviator strain (d) lack of coaxiality for stress direction and strain development



Figure 6. Simulated results of the behavior of Portaway sand in the drained state for $\alpha = 60^{\circ}$ (a) deviator stress-strain components (b) deviator stress-maximum main strain (c) bulk strain-deviator strain (d) lack of coaxiality for stress direction and strain development



Figure 7. Simulated results of the behavior of Portaway sand in the drained state for $\alpha = 90^{\circ}$ (a) deviator stress-strain components (b) deviator stress-maximum main strain (c) bulk strain-deviator strain (d) lack of coaxiality for stress direction and strain development

4. CONCLUSION

According to the above, the soil behavior is the function of the stress path applied to it. The results demonstrated the ability of Multi laminate theory to express soil behavior under mice and deformities.

As it can be seen in the loading directions of 0 and 90 degrees, we see the coaxiality of the main stress axes and main strain development. At 30 and 60 degrees, deviation from the coaxiality is observed at the beginning of loading due to plastic strains, but as the loading progresses, the tendency for coaxiality increases. In the 45 degrees' direction, we see a slight non-coaxiality. Therefore, this theory shows sensitivity to the smallest effects of stress or strain variation during simplicity and considers the effects of loading on its results. Also, given the availability of stress paths on the laminate, using this

theory we are able to predict the probable breakdown and the corresponding direction to an acceptable extent.

REFERENCES

- Hassanlourad, M., Salehzadeh, H., Shahnazari, H., 2008, Dilation and Particle breakage effects on the Shear Strength of Calcareous Sands based on energy Aspects, International Journal of Civil Engineering, Vol. 6, No. 2, pp 108-120.
- [2] Soltani-Jigheh, H., and Soroush, A., 2006, Postcyclic Behavior of Compacted Clay-sand Mixtures, International Journal of Civil Engineering, Vol.4, No. 3, pp 226-244.
- [3] Khoei, A.R., and Yadegari, S., and Anahid, M., 2006, Three Dimensional Modeling of Strain Localization in Cosserat Continuum Theory,

International Journal of Civil Engineering, Vol.4, No. 3, pp 176-192.

- [4] Soroush, A., and Koohi, Sh., 2004, Liquefaction-Induced Lateral Spreading- An Overview and Numerical Analysis, International Journal of Civil Engineering, Vol. 2, No. 4, pp 232-246. Alonso-Marroquín, F., Luding, S., and Herrmann, H. J. (2005). "Role of anisotropy in the elastoplastic response of a polygonal packing." Phys. Rev. E, 71(5), 051304.
- [5] Anand, L. (1983). "Plane deformations of ideal granular materials." J. Mech. Phys. Solids, 31(2), 105–122.
- [6] Cai, Y. (2010). "An experimental study of noncoaxial soil behaviour using hollow cylinder testing." Ph.D. thesis, Univ. of Nottingham, Nottingham, U.K.

[10]

- [7] Chu, J., Leong, W. K., Loke, W. L., and Wanatowski, D. (2012). "Instability of loose sand under drained conditions." J. Geotech. Geoenviron. Eng., 138(2), 207–216.
- [8] Chu, J., and Wanatowski, D. (2009). "Effect of loading mode on strain softening and instability behavior of sand in plane-strain tests." J. Geotech. Geoenviron. Eng., 135(1), 108–120.
 Gutierrez, M., and Ishihara, K. (2000). "Noncoaxiality and energy dissipation in granular materials." Soils Found., 40(2), 49–59.
- [9] Gutierrez, M., Ishihara, K., and Towhata, I. (1991). "Flow theory for sand during rotation of principal stress direction." Soils Found., 31(4), 121–132.
 10. Lade, P. V., Nam, Y., and Hong, W. P. (2009). "Interpretation of strains in torsion shear tests." Comput. Geotech., 36(1–2), 211–225.