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A Vendor Managed Inventory with FIFO and LIFO Plans

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Abstract

Nowadays, the stores always want to find a valid plan for perishable products as a challengeable concern for the management of perishable items. As known, regarding the perishable products which have a limitation for consumption, we cannot force the consumers to buy the products which are close to their shelf-life time. To address this challenge, this paper considers a hybrid of FIFO and LIFO strategies and its impact on the profits of the vendor managed inventory contracts which are very little discussion in the literature. Most importantly, this paper deals with an inventory system for perishable products stochastically with a new optimization model. The proposed problem computes the costs of the order and storage with two different holding costs based on the FIFO and LIFO strategies as well as the profits from selling of the products. With a maximizing optimization model, the goal is to find the optimal order for each store in per period. The model is solved by the dynamic programming with backward strategy. The results confirm the applicability and efficiency of the proposed model in this context and the high impact of products' shelf-life to use the FIFO or LIFO strategies in the stores.

Keywords

Vendor Managed Inventory, Perishable Products, FIFO, LIFO, Dynamic Programming

1. Introduction

With the increasing competitiveness of business in storing globalization, the vendor managed inventory is one of the most significant decisions in supply chain management (Fathollahi-Fard et al., 2019; Fleischmann et al., 1997). In terms of supply chain network design, it involves suppliers of raw materials, production plants (wholesalers), distribution centers (retailers) and customer zones (demand points) (Ijomah et al., 2004; Shi et al., 2011; Wang et al., 2011; Kim et al., 2013). It also cares about the number of items purchased, means of transportation, item flows between facilities, production

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planning and wholesaling, and storage levels with distributing and retailing the products (Ijomah et al., 2004; Shi et al., 2011). Because it has been suggested by experts that 80% of costs regarding supply chain are determined by facility locations and item flows between facilities (Wang et al., 2011; Kim et al., 2013; Giri & Sharma, 2015; Polotski et al., 2017) with focusing on the pricing and inventory decisions (Kim et al., 2013; Giri & Sharma, 2015), a better supply chain network should be strategically designed.

Distribution as one of important parts of supply chain network design, is characterized by the vendor managed inventory contract (Polotski et al., 2017; As' ad et al., 2019; Debo et al., 2005). In fact, the distribution is related to all activities with respect to purchasing of the products, transportation, and warehousing, financial costs for the network considering the purchasing conditions, order allocation to retailers in suitable place and time (Debo et al., 2005; Ferguson & Toktay, 2006; Jaber & El Saadany, 2009). Based on these needs and benefits from a well-designed of a distribution network, this study focuses on the role of stores to focus on the inventory management under the vendor managed inventory contract for perishable products with both FIFO[1] and LIFO[2] strategies.

The inventory management has a crucial role in reducing the system costs and increasing customer satisfaction through increasing responsiveness and also it can reduce the bullwhip effects caused by fluctuating inventory levels (Hasanov et al., 2012; Gan et al., 2017). Thereby, this paper utilizes a vendor managed inventory policy for the proposed distribution and inventory model because of its efficiency in inventory control management (Taleizadeh & Moshtagh, 2019; Kenné et al., 2012; Stock & Broadus, 2006). This inventory model is used to improve the agile supply chain performance in a way, all levels benefit (Disney & Towill, 2003; Darwish & Odah, 2010; Schenck & McInerney, 1998). In this strategy, a set of the manufacturer, remanufacturer, warehouses, and the retailer are integrated (Angulo et al., 2004; Diabat, 2014). Retailer shares its own customer's demand and inventory position data to the vendor, to make the appropriate decisions on the order size (Jaber, 2006; Wright, 1936). Applying a proper design in the vendor managed inventory strategy decreases the inventory costs, response time to customer demands, and enhances a good collaboration at the different levels of chains (Govindan et al., 2015; Guide & Van Wassenhove, 2009; Van der Laan, 2019).

Since one of the key issues in this paper is to determine the replenishment rate and order quantity, it is necessary to adopt good inventory management (Hajiaghaei-Keshteli & Fathollahi-Fard, 2019; Sahebjamnia et al., 2018; Fathollahi-Fard & Hajiaghaei-Keshteli, 2018). Therefore, this paper integrates the distribution network with the vendor managed inventory model for perishable products. The vendor contains a manufacturer and remanufacturer and is responsible to make decisions on determining the order size, the number of shipments, and acceptance quality level of returned products (both manufactured and remanufactured products) (Hajiaghaei-Keshteli & Fathollahi-Fard, 2019; Safaeian et al., 2019; Bazan et al., 2015a; 2015b). It should be kept in mind, ordering costs and vendor's holding cost is in the part of its own costs while the retailer should pay for its stock costs. Besides, along with the production process, defective products are inevitable (Bazan et al., 2017; Qiu et al., 2018; Marchi et al., 2019a; Jaber & Bonney, 2003). However, this study does not assume the defective products for plan of distribution in perishable items. As all products are perishable like milk and yogurt, this study contributes also in FIFO and LIFO accounting methods (Jaber, 2006). These issues are used in the inventory management especially for perishable products with regards to the financial concerns. They are utilized to manage the cost assumptions related to the inventory, stock repurchases and different other accounting goals in a distribution network.

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[1] First-In, First-Out (FIFO)

[2] Last-In, First-Out (LIFO)

Concerning the above considerations, we present a novel vendor managed inventory contract for the distribution of the perishable products. These need to be addressed that this paper follows the most assumption of the proposed model by (Fathollahi-Fard et al., 2019; Taleizadeh & Moshtagh, 2019). Both investigated a multi-echelon supply chain network model with a dependable return rate on the acceptance quality level of used products also they examined the inventory control planning under the vendor managed inventory policy with considering the defective products and rework process. In the real problem, the proportion of defective items is a random variable with a probability density function. In comparison with them, this study focuses on the role of stores to focus on the inventory management under the vendor managed inventory contract for perishable products with both FIFO and LIFO strategies. It should be noted the use of these strategies for perishable products is established by the recommendations of Crama et al. (2018) who considered a routing-inventory model for perishable products.

In conclusion, this study establishes following contributions to the literature:

- A vendor managed inventory contract for the perishable products, is considered.
- An inventory system is stochastically developed for the special products.
- The plan of stores for perishable products is to use both FIFO and LIFO strategies.
- A stochastic dynamic programming approach with the use of backward decision tree is proposed for the developed model.

This paper follows five sections. Section 2 reviews the recent advances and important papers in the literature and divides it into two subsections. Section 3 explains the framework of the proposed problem and formulates it. Section 4 establishes the solution method and Section 5 provides a comprehensive analysis based on the parameters of the model and the quality of solutions. Finally, conclusion and future remarks are conducted in Section 6.

2. Literature Review

The literature of the inventory models for distribution networks, is so much rich. To identify the research gaps, we study the relevant works based on six criteria including inventory control policy (which is traditional or the VMI), variety of the products (which is single or multi-product), the time horizon (which is finite or infinite), the possibility of shortage, the product perishability, the imperfect product and multi-delivery consideration. In this regard, Table 1 provides a review on the distribution network studies. Having an updated literature review, only the most relevant papers contributing to the inventory systems from 2006 till now, are studied.

The distribution network is an active research topic and many studies combined it with inventory strategies as well as the perishable products. The distribution network is a part of the closed-loop supply chain (CLSC) problem which is comprehensively reviewed by Govindan et al. (2015), Guide & Van Wassenhove, (2009), and Van der Laan, (2019) as well as some recent advances about CLSC may be found at (Hajiaghaei-Keshteli & Fathollahi-Fard, 2019; Sahebjamnia et al., 2018; Fathollahi-Fard & Hajiaghaei-Keshteli, 2018; Safaeian et al., 2019). Although the CLSC problem is studied in various fields, here concerns the vendor managed inventory (VMI) contract. To show the high impact of the VMI policy in comparison with the traditional inventory systems, for example, Bazan et al. (2015a) investigated the VMI policy and classical inventory control policy for a manufacturing-remanufacturing reverse logistics to minimize the total costs of the system by determining the number of manufacturing and remanufacturing batches per cycle, the number of times a product remanufactured and the size of manufacturing batches. In another similar paper, Bazan et al. (2015b) considered a single-vendor single-buyer system under the town inventory control policies. The first one is classical inventory policy and another one is VMI policy. In both policies, reducing the energy used in production and transportation is found priority to minimize the system cost. The same authors, Bazan et al. (2017) presented a single-manufacturer single-retailer closed-loop supply chain model under the VMI policy with a consignment stock contract (CS). As a result, VMI-CS is more economical than the classical inventory policy but it's not necessarily the environmental choice. In another paper, Qiu et al. (2018) introduced closed loop production rooting problem with remanufacturing and reverse logistics activities under the VMI contract. They provided a comparison of system costs concerning the different remanufacturing parameters.

The recent studies are generally a multi-product VMI model with the multi-delivery supposition. For example, Marchi et al. (2019a) offered a two-level (vendor-buyer) supply chain models considering two both classical and vendor-managed inventory with consignment stock (VMI-CS). In another study, Taleizadeh & Moshtagh (2019), developed a CLSC model under the VMI contract considering the manufactured and remanufactured products with different quality. They considered the acceptable quality level of returned items as a variable that return rate depends on it. Because of the simple mathematical form of learning curve produced by Wright (1936), most researchers have applied this theory to a diverse set of sciences. Wright (1936) found that the learning rate reduces production time thereby system costs will diminish. This concept is reflected in the lot-sizing problem. Jaber & Bonney (2003; 2011) provided a wide review of the learning curve in lot sizing in the literature. In 2011, Jaber & El Saadany, investigated an inventory in reverse logistics with regard to the learning effects to better manage the inventory and utilize the available capacity. Also, the learning effects occur in both manufacturing and remanufacturing processes. Jaber et al. (2011) proposed a mathematical model of an integrated supply chain with continuous improvement in the production process by applying the learning curve. Later, Zanoni et al. (2012) considered a VMI policy for a two-level supply chain. The learning effects in the production process give an opportunity to the vendor to apply and compare different consignment agreement policies. Khan et al. (2014), developed an integrated supply chain model under the learning curve effects in the production process. Also in their investigation defective products occur due to human errors. They assumed multi-delivery Economic Production Quantity (EPQ) policy for the vendor and traditional Economic Order Quantity (EOQ) policy for the buyer. Manna et al. (2017) extended an EPQ model with allowable shortages. In their model, imperfect items are reworked or disposed of then to reduce the defective items, the learning effect is considered in the production process. In another different paper by Crama et al. (2018), a routing-inventory system was developed and addressed by a various heuristics. They recommended that the use of both FIFO and LIFO strategies is highly useful to improve the performance of their model.

In an infinite time horizon, the learning can be seen to improve the efficiency of the distribution networks. For example, Marchi et al. (2019b) investigated the learning effect in energy efficiency which is an essential factor in many manufacturing companies. Therefore, they proposed a lot-sizing problem to illustrate the interaction between learning in production and energy efficiency directly and indirectly and also an appropriate decision about the lot size quantity. Walid et al. (2019) applied the learning effects in the mean and variance of non-conforming items which considered as a random variable. Therefore, they decrease under the effect of the learning process. Finally, Chen et al. (2019) considered a firm (e.g., retailer) selling a single nonperishable product over a finite-period planning horizon to make pricing and ordering decisions based on observed demand data.

In conclusion, based on the characteristics of aforementioned papers as summarized in Table 1, following findings can be concluded:

Table 1. Review on the relevant works contributing to the inventory systems

Studies	Inventory Control Policy		Variety of Product		Time Horizon		Shortage		Perishable Products	Multi-Delivery
	Traditional	VMI	Single	Multi	Finite	Infinite	No Shortage	Lost Shortage		
Jaber et al. (2010)	✓		✓			✓	✓			✓
Jaber & El Saadany, (2009)	✓		✓			✓	✓			
Zanoni et al. (2012)		✓	✓		✓		✓			✓
Khan et al. (2014)	✓		✓		✓		✓			✓
Bazan et al. (2015a)	✓		✓			✓	✓		✓	✓
Bazan et al. (2015b)	✓	✓	✓			✓	✓			✓
Taleizadeh & Moshtagh, (2019)		✓	✓			✓	✓		✓	✓
Bazan et al. (2017)	✓	✓		✓		✓	✓			✓
Manna et al. (2017)	✓		✓		✓			✓		
Qiu et al. (2018)	✓		✓		✓		✓			
Marchi et al. (2019b)	✓		✓			✓	✓			
Walid et al. (2019)	✓		✓		✓			✓		✓
Marchi et al. (2019a)		✓	✓			✓	✓			✓
Chen et al. (2019)		✓		✓		✓		✓		
Crama et al. (2018)		✓		✓	✓		✓		✓	
This study		✓		✓	✓		✓		✓	✓

- Recent studies are mainly interested to use the benefits of VMI policies to address their problems.
- The multi-product inventory systems in our field are considered in only three studies Bazan et al. (2017), Chen et al. (2019), and Crama et al. (2018).
- The time horizon may be finite or infinite in the literature. This study uses a multi-period model which is finite time horizon.
- Shortage is not allowed by many studies like this paper.
- The development of inventory systems for the perishable products is rarely contributed in the literature like this paper.

- Simultaneous consideration of FIFO and LIFO strategies for perishable products is not studied as far as we know.

Regarding these findings, especially the last one, it goes without saying that it is much more difficult to control the stock of perishable products due to the low desire to buy the old products. Therefore, buyers cannot be required to purchase older products. In the case of these products, the nature of the objective function, revenues and costs change, and the strategy for determining the optimal order quantity changes. Therefore, in this paper, the following highlights have been done to improve inventory control policy:

- Products are divided into fresh and old categories.
- A hybrid inventory policy combining FIFO and LIFO is developed to satisfy the demand of consumers.
- The proposed model reduces the price of old products due to the remaining period of the shelf-life date in order to increase the desire to buy for consumers.
- The proposed model increases the cost of holding for old products due to the remaining time from the shelf-life date. It confirms the more difficulty of the storage conditions for the old products.

3. Model Description

The current study is investigating a VMI model composed of a set of stores. Indeed, this model is a continuation of the previous works in this research area Giri & Sharma, (2015), Taleizadeh & Moshtagh, (2019), Crama et al. (2018). These studies were carried out assuming the minimum acceptable quality level for returned products to be remanufactured. This study follows a same path but does not contribute to the returned and remanufactured products. In comparison with Crama et al. (2018) specifically, this study proposes a distribution network for perishable products. Although Crama et al. (2018) considers only FIFO plan for all products, this study makes a different between old and new products. The fresh products are planed firstly by FIFO policy and then the old ones are distributed by the LIFO policy. This is the main innovation and significant contribution of this paper. Generally, we follow the assumption operated in below:

The proposed problem computes the costs of the order and storage as well as the benefits from selling the products to maximize the profits of the stores. This distribution network model contains the VMI policy and a plan to use both FIFO and LIFO strategies (Fathollahi-Fard et al., 2019b; Alvarez et al., 2020; Fathollahi-Fard et al., 2020). Moreover, the production process and the remanufacturing process are not considered in this model. Defective products will be not allowed to distribute by the stores. These factors can also improve the satisfaction levels of the customers (Li et al., 2020). At last but not least, this study applies the inventory systems for perishable products as similarly studied in Crama et al. (2018).

To define the proposed problem briefly, let's assume that we have one type of product which has seven days as its maximum shelf-life date (L=7). In this regard, the items which have 4 to 7 days as the shelf-life date, are the fresh products (T'=4). The rest of items which have 1 to 3 days, are the old products. In this example, the inventory status is defined as (x_1=85; x_2=37; x_3=128; x_4=210; x_5=90; x_6=290) with the probability of [0.2, 0.3, 0.2, 0.3]. To satisfy the demand which is stochastic, three cases are assumed in the model:

i. The demand is much bigger than the current inventory status and the order value.
ii. The demand is maximally fit with the fresh products.

iii. The demand is more than the fresh products and lower than the inventory status.

With regards to one of aforementioned conditions, the model aims to find the optimal order and the maximum profits for the stores (Eqs. (1) and (4)). This goal is optimized with regards to the FIFO and LIFO policies. Fig. 1 shows the decision tree for the demand of stores and accordingly, an optimal answer to address the customers' demand is found for these customers [x_4, x_5, x_6, x_3, x_2, x_1], respectively. It should be noted the final optimal solution at the end of the second period is [128, 10, 0, 180, 60, 380].

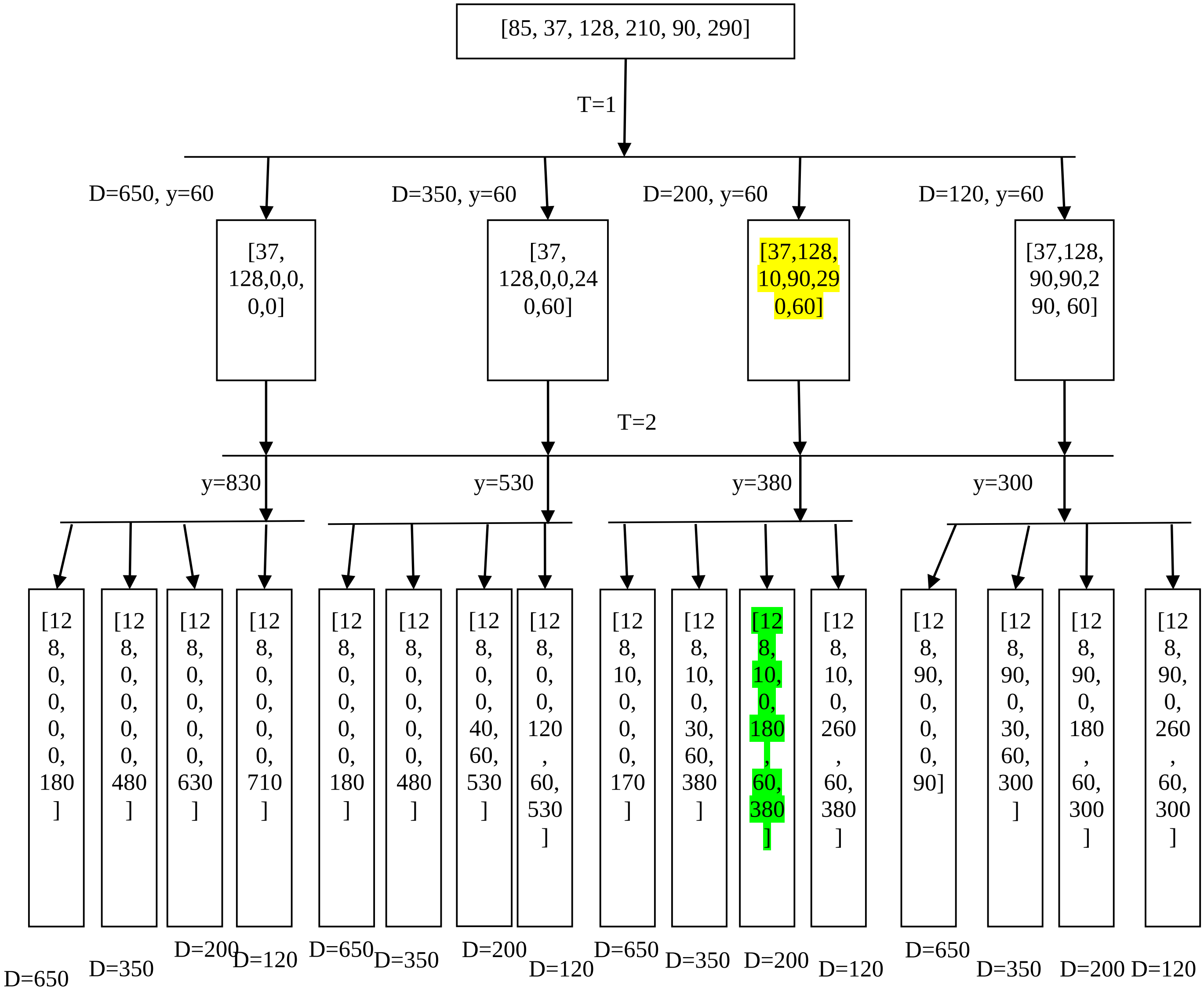


Fig. 1. Numerical example (the optimal solution is highlighted by green color)

To assess the decision tree in Fig. 1, note that the nodes represent the inventory at the beginning of the period and the branches represent different demand scenarios with a distribution of specific probabilities. In this example, the beginning node is [85, 37, 128, 210, 90, 290]. These numbers indicate the status of inventories at the beginning of the period with an expiration date of one to six days, respectively. Managers in each store decide on how much to order by checking inventory at the beginning of the period to maximize total store profits at the end of the period. Note that the orders will have an expiration date of 7 days.

In this example, products with an expiration date of 4 to 7 days are considered as new products. Conversely, the old products have an expiration date of one to three days. Accordingly, the first period (T = 1) includes four demand scenarios in order to respond to the customers. Depending on the occurrence of each case of demand scenario and order quantity, the new products are used first and then, old products will be used if needed. For example, if the number of requests is 650 and the number of orders is 60, the order of response will be [x_4, x_5, x_6, x_7, x_1, x_2, x_3] with the values [210, 90, 290, 60, 128, 37, 85]. In this case, the number of new products, i.e., [x_4, x_5, x_6, x_7] meets the demand and there is no need to use the old products. This procedure is done for other demand nodes in three other scenarios.

In the second period (T= 2), to achieve the balance at the beginning of the period, after responding to the customer, the balance of the first period is updated. Note that the product that has been expired in the first period, cannot be used in the second period. As given in the first period, the number of demand was 650 and the order was 60. The updated inventory at the beginning of the second period will be [37, 128, 0, 0, 0, 0]. As four demand scenarios are existed, 16 demand scenarios are existed in the second period. This procedure will be done for other demand scenarios.

Finally, to establish the mathematics of the proposed model, the notations are:

Indices:

i	Index of stores
t	Index of periods
k	Index of remaining shelf-life of products

Parameters:

N	Number of stores
P	Selling price of each unit of the product
α	Rate of reduction of the purchasing price of each unit for old products
L	The maximum shelf-life of the products
h	Holding cost per unit
D_{ti}	Demand of store i in period t (This parameter is generated randomly)
T	Length of the planning horizon
c_i	Capacity of store i
A_{ti}	Estimated cost-to-serve assignment of store i in period t
β	Rate of growth of holding cost for old products
$P(D_{ti}=d)$	Probability function of the demand of store i in period

Decision variable:

y_{ti}	Delivery quantity in period t for the store i
T'	A part of shelf-life date to define the fresh products and the old products

The inventory status in each period for each store is: $x_{ti} = (x_1, x_2, \dots x_{L-1})_{ti}$

For example, x_1 means the number products which has one day as their shelf-life date. As such, $(x_k)_{ti}$ is the number of products which have k days as the shelf-life dates.

In this regard, the expected profit for $T+1$ period is:

$$f^*_{T+1,i}\big((x_1, x_2, \dots x_{L-1})_{T+1,i}\big) = (\frac{a}{2})I_{T+1,i} \tag{1}$$

Where α is the rate of reduction of the purchasing price of each unit for old products which is computed by the following formula:

$$\alpha = \frac{P}{2\times(T\prime-1)} \tag{2}$$

Regarding the above equation, it is assumed that the old products for their last day of the shelf-life date will be bought as a fraction for its half of initial price. In addition, the rate of β for the growth of holding cost for the old products is calculated by the following formula:

$$\beta = \frac{0.2 * h}{(T' - 1)} \tag{3}$$

From Eq. (3), it is assumed that for old products in the last day of the shelf-life date, a fraction of 20 percent of holding cost will be considered to compute the extra cost. Finally, the objective of our model ($f_{ti}(x_{ti}, y_{ti})$) based on each period and the level of distribution is computing all the costs with regards to the inventory costs of the network:

$$f_{ti}(x_{ti}, y_{ti}) = -A_{ti}|(y_{ti} > 0) - ay_{ti} - h\sum_{K=T'}^{L-1}(x_k)_{ti} - \sum_{k=1}^{T'-1}(h + \beta(T' - k\,)^+)\,(x_k)_{ti} \tag{4}$$

$$+pr(D_{ti} > I_{ti} + y_{ti})\left[p\,(\sum_{K=T'}^{L-1}(x_k)_{ti} + y_{ti}) + \sum_{k=1}^{T'-1}(P - \alpha(T' - k\,)^+)(x_k)_{ti} + \right.$$

$$\left. f^*_{t+1,i}(0,0,\dots,0)\right] + \left(\sum_{d=0}^{I^f_{ti}+y_{ti}} pr\,(D_{ti} = d)(pd + f^*_{t+1,i}(x'_{t+1,i}))\right) +$$

$$\sum_{d=I^f_{ti}+y_{ti+1}}^{I_{ti}+y_{ti}} pr\,(D_{ti} = d)\,(p(I^f_{ti} + y_{ti}) + \sum_{k=T''+1}^{T'-1}(p - \alpha(T' - k)^+)(x_k)_{ti} +$$

$$f^*_{t+1,i}(x''_{t+1,i}) + (d - (I^f_{ti} + y_{ti}) - \sum_{k=T''+1}^{T'-1}(x_k)_{ti}\,)\,(p - \alpha(T' - T'')^+)))$$

Following, T'' is indexed by k which is valid in Eq. (5):

$$\sum_{k=T''}^{T'-1}(x_k)_{ti} \geq d - (\,I^f_{ti} + y_{ti}) \tag{5}$$

The fresh products are planed firstly by FIFO policy (Crama et al., 2018). The products which are close to the shelf-life date are planned with LIFO policy (Li et al., 2020). In this regard, following equations are provided:

$$x_{ti} = (x_{T'}, x_{T'+1}, x_{T'+2}, \dots, x_{L-1}, x_L, x_{T'}, x_{T'-1}, x_{T'-2}, \dots, x_1)_{ti} \tag{6}$$

$$(x'_k)_{t+1,i} = \begin{cases} ((x_{k+1})_{ti} - (d_{ti} - \sum_{L=T'}^{k+1}(x_L)_{ti}))\,; & k = T' - 1, \dots L - 1 \\ (x_k)_{ti} \quad ; & k = 1,2,\dots,T' - 2 \end{cases} \tag{7}$$

$$(x''_k)_{t+1,i} = \begin{cases} ((x_{k+1})_{ti} - (d_{ti} - I^f_{ti} - y_{ti} - \sum_{L=k+1}^{T'-1}(x_L)_{ti})^+)^+; & k = 1,2,\dots,T' - 2 \\ 0 \quad ; & k = T' - 1, \dots, L - 1 \end{cases} \tag{8}$$

In addition, consider:

$$I_{ti}^{f} = \sum_{K=T'}^{L-1} (x_k)_{ti} \tag{9}$$

$$I_{ti}^{d} = \sum_{k=1}^{T'-1} (x_k)_{ti} \tag{10}$$

$$I_{ti} = I_{ti}^{f} + I_{ti}^{d} \tag{11}$$

Final model is:

$$\max \sum_{i=1}^{N} \sum_{t=1}^{T} f_{ti}\,(x_{ti}\,, y_{ti}) \tag{12}$$

s.t.

$$f *_{ti}\,(x_{ti}) = \max f_{ti}(\,x_{ti}, y_{ti}) \tag{13}$$

$$0 \le y_{ti} \le c_i - I_{ti} \tag{14}$$

4. Solution Method

This study solves the proposed model given in Eqs. (12) to (14) by a Stochastic Dynamic Programming (SDP) approach. The SDP showed its high performance solutions to determine the optimal order in several studies such as Walid et al. (2019), and Crama et al. (2018). This solution method is based on a backward strategy to consider all possible solutions numerically. In this regard, the probability of demand occurring during this and subsequent periods, and inventory, determines the optimal order quantity. The backward strategy and problem solving works in such a way that in each period, the inventory of that period is examined and according to the inventory and the probability of demand, the optimal order amount (y_{ti}) is determined.

The SDP also uses a decision tree is needed to consider all possible cases numerically. The decision tree is a tool for decision-making when we have a number of stochastic scenarios. To form the decision tree Crama et al. (2018), regarding the initial inventory, we must determine the order amount for each probability of the demand from the first period with the initial inventory, and according to the demand, the allocation is done for each store. This decision tree has $I \times D^T$ size (see Fig. 1 as an example). To implement this algorithm, following rules should be met:

- The policy for answering the customers demand is started with the fresh products by FIFO policy. Then, the old products are allocated by LIFO policy.
- The remaining inventory at the end of the period is sold at the half of the purchase price for the products.

Finally, the main steps of our proposed SDP for the developed model are:

Step 0: Let i=1 and t=1.

Step 1: Evaluate the nodes which are not checked in the decision tree.

Step 2: If the list of the nodes from the decision tree is filled, go to Step 6.

Step 3: For this store, the final order at the end of inventory status period and the sequence of distribution should be computed.

Step 4: For each demand scenario in this store, repeat the procedure in Step 3 to finalize all the nodes and to fill the list for all the demand scenarios.

Step 5: Update the period from t to t+1. If there is still a demand scenario, go to Step 4. If the number of period is ended to Step 6.

Step 6: Update the store from i to i+1. If the store i+1 is available, go to Step 2 for the list of decision tree. Otherwise, compute the expected total profit in all the stores from the latest order and inventory status to the first one.

5. Computational Results

To solve the model and to provide the numerical example, the model given in Section 3 is solved by a SDP approach with the backward strategy as provided in Section 4. In this regard, the model is addressed and some sensitivity analyses are done to address the problem solving. Since there is no innovation in the concept of the solution algorithm, more details about this method are referred to the literature, especially (Jaber & Bonney, 2003; 2011; Jaber & El Saadany, 2011; Jaber et al., 2010). Notably, the numerical instance is taken from Crama et al. (2018) to do the tests. Moreover, the Matlab software is used to solve the Eqs. (12) and (14). It goes without saying that the proposed SDP with the use of decision tree is only available for small sizes. Note that the model is coded in an Intel Pentium Quad Core 3 GHz pc with 4 GB of ram.

In this regard, the inputs of the model are as follows: the price of each product is 80$; three periods are considered with the holding cost of 10$. Four demand scenarios with the probability of 0.2, 0.3, 0.2 and 0.3 are considered for the amount of the demand including 120, 200, 350 and 650 respectively. The initial inventory is zero and the capacity of each store is 1000 products. The inventory statuses of the vendor managed inventory contract is [x_1=85, x_2=37, x_3=128, x_4=210, x_5=90, x_6=290].

Based on this numerical example, the optimum total profit per unit time can be calculated by substituting these equations in Eq. (12) resulting in Table 2. Using the figures in Table 2, we can see that the total profit has been increased per period as the number of markets has been increased.

Table 2. Analysis of the results

i	T	Total Profit
1	1	79200
1	2	84400
1	3	89600
1	4	102080
2	1	207550
2	2	226400
2	3	231200

The sensitivity analysis is done on three important parameters including α, β and *T'*. In our SDP, we have considered the rate of α given in Eq. (2) as 13.3, β given in Eq. (3) as 0.66 and *T'* as 4. To show the impact of these parameters, four inputs are given and then, the behavior of the objective function given in Eq. (12) as well as the computational time of the SDP are analyzed. The results are given in Table 3, 4, and 5 as well as the behavior of the objective and the computational time is shown in Fig. 2, 3, and 4.

Table 3. The sensitivity analysis on α

No. of Cases	α	Objective Function ($)	Computational Time (Second)
C1	10	118400	202
C2	13.3	98600	205
C3	16	73800	208

C4	20	59300	209

Table 4. The sensitivity analysis on β

No. of Cases	β	Objective Function ($)	Computational Time (Second)
C1	0	73400	222
C2	0.66	69200	217
C3	1	65400	213
C4	10	38800	211

Table 5. The sensitivity analysis on T'

No. of Cases	*T'*	Objective Function ($)	Computational Time (Second)
C1	1	116800	201
C2	2	113900	207
C3	3	101500	213
C4	4	99800	218

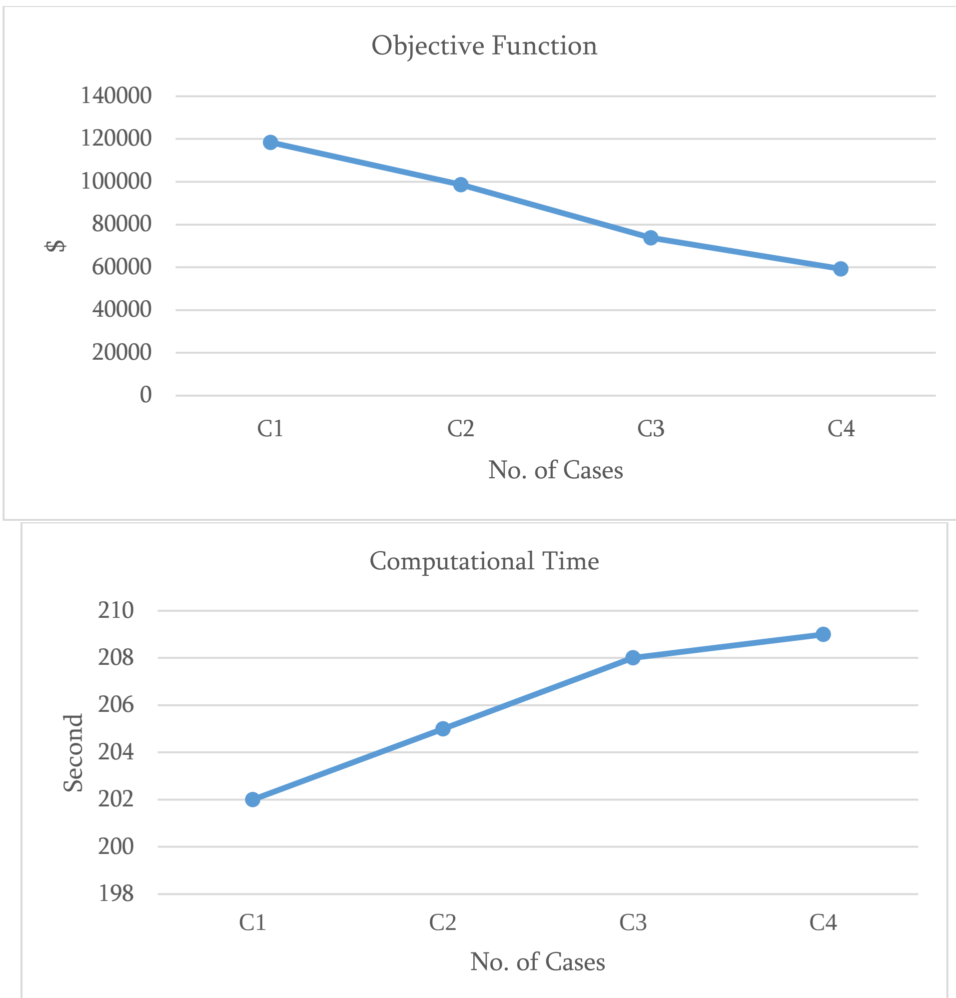
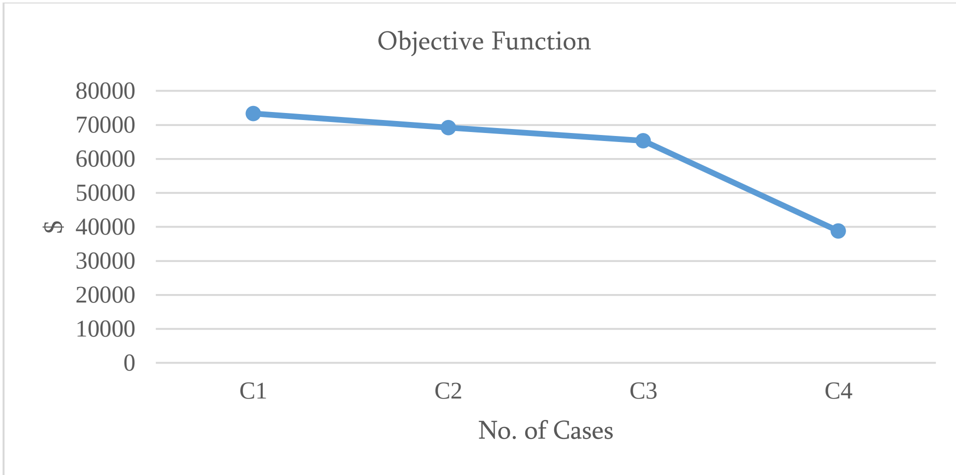


Fig. 2. The sensitivity analysis on α



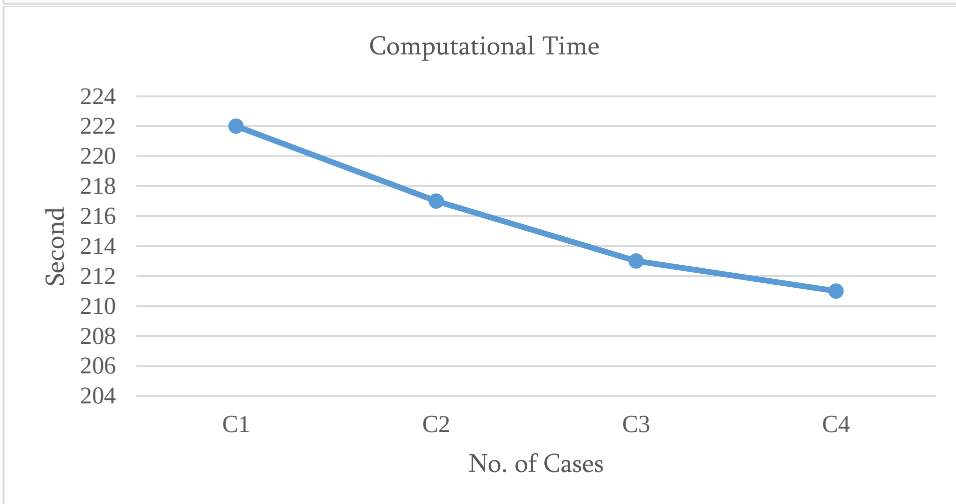
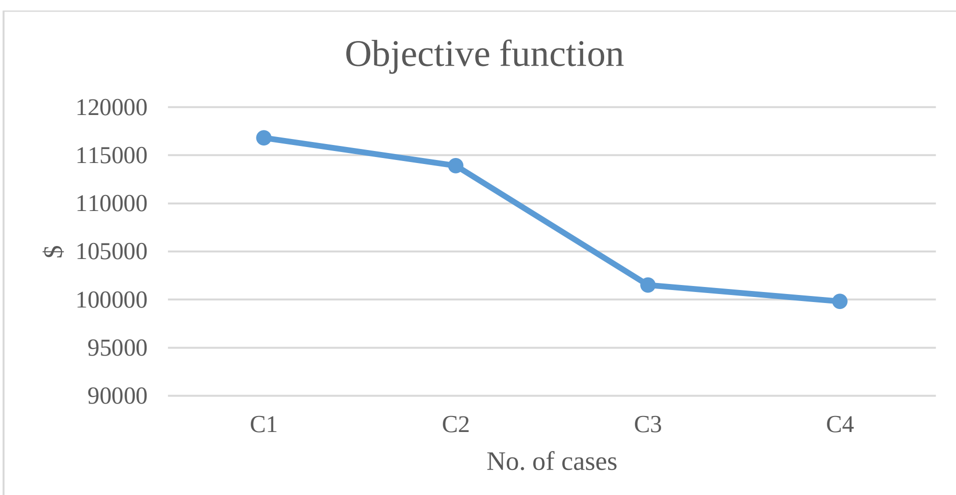


Fig. 3. The sensitivity analysis on β



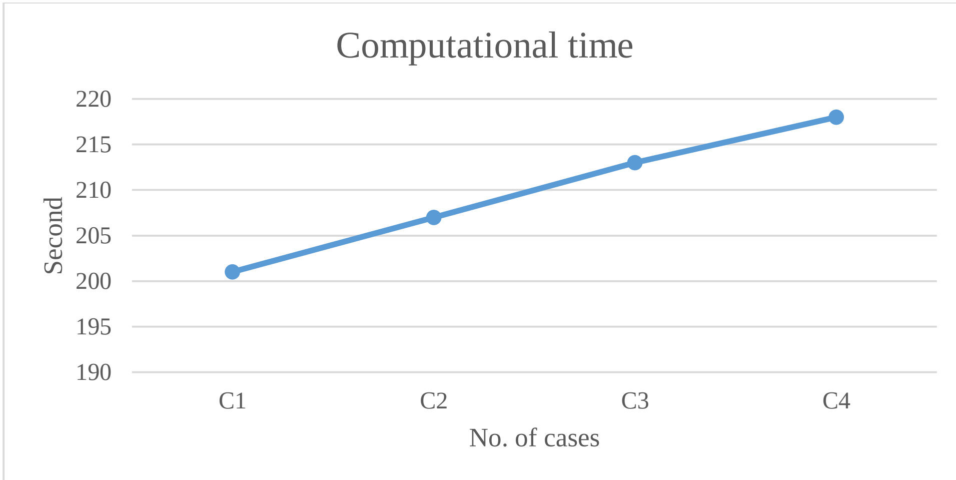


Fig. 4. The sensitivity analysis on T'

First of all, using the figures in Table 3 and the behavior of the objective and the computational time depicted in Fig. 2, shows that the parameter α is directly affected on the objective function. This parameter can reduce the profits in the objective function. In addition to its impact on the model, an increase in this factor, increases the computational time of the SDP for solving the model.

Secondly, using the figures in Table 4 and the behavior of the objective and the computational time of SDP depicted in Fig. 3, reveals that a well-tuned value of this factor is highly important as it reduces the total profits. This factor should be selected a low value for the managers if they want to improve the performance of their inventory system. This factor also reduces the computational time of SDP. An increases in this factor decreases the process time.

Lastly, using the figures in Table 5 and the behavior of the objective and the computational time of SDP method depicted in Fig. 4 confirms that the use of FIFO policy for the fresh products increases the profits of the model. If we give more time to do the FIFO policy and then apply the LIFO policy, it will be useful to sell the products which are close to its shelf-life date. For example, from one to two days to change the inventory policy, the profits of the model show a high growth as compared with other cases. It should be noted if T' equals to one, the model cannot make a difference between the FIFO and LIFO strategies and the model is the same as the basic one proposed in Crama et al. (2018). It goes without saying that an increases in this parameter increases the process time of the SDP algorithm uniformly.

What we can see is that the profits of the stores are generally more sensitive to the changes in input data for the case of stores complexity and shelf-life of products in contrast to the case of product's price regarding the FIFO and LIFO policies.

6. Conclusion

In this paper, concerning to recent changes in the distribution networks liked with the vendor managed inventory models, a novel VMI policy for a set of stores was developed. To identify the contributions of the proposed model and to distinguish it from the previous works, several extensions have been added. In the real problem, the proportion of perishable products with regards to the fresh or old ones are not well-studied enough. Although some researchers deal with this assumption in their distribution planning, not enough attention has been paid to this. To fill this gap, we assume an integrated policy to consider both FIFO and LIFO policies for perishable items. The fresh products are planned with the FIFO policy and other products are scheduled by the LIFO policy to do the distribution role by stores. Therefore, the significant contribution of this paper is to propose a VMI including a set of stores to focus on the inventory management for perishable products with both FIFO and LIFO strategies. The effects of these factors were analyzed via a set of sensitivities. The model was solved by the SDP based on the backward strategy. Finally, the results also confirm the applicability and high efficiency of the proposed model in real concepts of the VMI.

For future studies, there are many insightful recommendations. Extending the proposed problem by adding sustainability factors such as the green emissions, suppliers' risk and satisfaction levels. More broadly, employing novel heuristics and metaheuristics is another good continuation of this study.

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